9 Arithmetic: Fractions and Percentages

9.1 Revision of Operations with Fractions

In this section we revise the basic use of fractions.

Addition

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

Note that, for *addition* of fractions, in this way both fractions must have the *same denominator.*

Multiplication

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Division

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

$$= \frac{a \times d}{b \times c}$$

Example 1

Calculate:

(a) $\dfrac{3}{5} + \dfrac{4}{5}$

(b) $\dfrac{3}{7} + \dfrac{1}{3}$

Solution

(a) $\dfrac{3}{5} + \dfrac{4}{5} = \dfrac{3+4}{5}$

$\qquad\qquad = \dfrac{7}{5}$

$\qquad\qquad = 1\dfrac{2}{5}$

(b) $\dfrac{3}{7} + \dfrac{1}{3} = \dfrac{9}{21} + \dfrac{7}{21}$ (*common* denominator $= 21$)

$\qquad\qquad = \dfrac{16}{21}$

Example 2

Calculate:

(a) $\dfrac{3}{4}$ of 48

(b) $\dfrac{3}{5}$ of 32

Solution

(a) $\dfrac{3}{4}$ of 48 $= \dfrac{3}{4} \times 48$

$= \dfrac{3 \times 48}{4}$

$= 36$

(b) $\dfrac{3}{5}$ of 32 $= \dfrac{3}{5} \times 32$

$= \dfrac{3 \times 32}{5}$

$= \dfrac{96}{5}$

$= 19\dfrac{1}{5}$

Example 3

Calculate:

(a) $\dfrac{3}{4} \times \dfrac{3}{7}$

(b) $1\dfrac{1}{2} \times \dfrac{2}{5}$

Solution

(a) $\dfrac{3}{4} \times \dfrac{3}{7} = \dfrac{3 \times 3}{4 \times 7}$

$= \dfrac{9}{28}$

(b) $1\dfrac{1}{2} \times \dfrac{2}{5} = \dfrac{3}{2} \times \dfrac{2}{5}$ or $1\dfrac{1}{2} \times \dfrac{2}{5} = \dfrac{3}{{}_1\cancel{2}} \times \dfrac{\cancel{2}^{1}}{5}$

$= \dfrac{6}{10}$ $= \dfrac{3}{5}$

$= \dfrac{3}{5}$

Example 4

Calculate:

(a) $\dfrac{3}{7} \div \dfrac{3}{4}$

(b) $1\dfrac{3}{4} \div \dfrac{4}{5}$

Solution

(a)

$$\dfrac{3}{7} \div \dfrac{3}{4} = \dfrac{3}{7} \times \dfrac{4}{3}$$

$$= \dfrac{12}{21}$$

$$= \dfrac{4}{7}$$

or

$$\dfrac{3}{7} \div \dfrac{3}{4} = \dfrac{{}^1\cancel{3}}{7} \times \dfrac{4}{\cancel{3}_1}$$

$$= \dfrac{4}{7}$$

(b)

$$1\dfrac{3}{4} \div \dfrac{4}{5} = \dfrac{7}{4} \div \dfrac{4}{5}$$

$$= \dfrac{7}{4} \times \dfrac{5}{4}$$

$$= \dfrac{35}{16}$$

$$= 2\dfrac{3}{16}$$

Exercises

1. Calculate:

(a) $\dfrac{1}{7} + \dfrac{4}{7}$

(b) $\dfrac{3}{8} + \dfrac{7}{8}$

(c) $\dfrac{1}{9} + \dfrac{7}{9}$

(d) $\dfrac{3}{10} + \dfrac{1}{10}$

(e) $\dfrac{7}{13} + \dfrac{9}{13}$

(f) $\dfrac{6}{7} + \dfrac{5}{7}$

(g) $\dfrac{5}{7} - \dfrac{3}{7}$

(h) $\dfrac{7}{9} - \dfrac{4}{9}$

(i) $\dfrac{11}{13} - \dfrac{6}{13}$

2. Calculate:

(a) $\dfrac{1}{2} + \dfrac{1}{3}$

(b) $\dfrac{1}{5} + \dfrac{1}{7}$

(c) $\dfrac{1}{4} + \dfrac{1}{5}$

(d) $\dfrac{2}{3} + \dfrac{1}{2}$

(e) $\dfrac{7}{8} + \dfrac{3}{10}$

(f) $\dfrac{3}{4} + \dfrac{4}{5}$

(g) $\dfrac{3}{7} + \dfrac{2}{3}$

(h) $\dfrac{4}{9} + \dfrac{2}{3}$

(i) $\dfrac{1}{4} + \dfrac{5}{8}$

3. Calculate:

(a) $1\dfrac{1}{2} + 2\dfrac{1}{2}$

(b) $3\dfrac{3}{4} + 4\dfrac{1}{4}$

(c) $2\dfrac{3}{5} + 3\dfrac{1}{5}$

(d) $3\dfrac{1}{3} + 1\dfrac{1}{2}$

(e) $3\dfrac{4}{5} + 2\dfrac{3}{5}$

(f) $5\dfrac{4}{7} + 3\dfrac{4}{7}$

(g) $4\dfrac{3}{4} + 2\dfrac{5}{8}$

(h) $4\dfrac{2}{7} + 3\dfrac{1}{3}$

(i) $2\dfrac{5}{9} + 3\dfrac{2}{3}$

4. Calculate:

(a) $2\dfrac{1}{2} - 1\dfrac{1}{2}$

(b) $4\dfrac{3}{4} - 3\dfrac{1}{4}$

(c) $2\dfrac{3}{8} - 2\dfrac{1}{4}$

(d) $4\dfrac{5}{7} - 3\dfrac{6}{7}$

(e) $3\dfrac{5}{8} - 1\dfrac{7}{8}$

(f) $4\dfrac{1}{3} - 3\dfrac{1}{2}$

(g) $2\dfrac{2}{3} - 1\dfrac{1}{9}$

(h) $5\dfrac{3}{7} - 2\dfrac{1}{2}$

(i) $4\dfrac{1}{4} - 2\dfrac{2}{3}$

5. Calculate:

(a) $\dfrac{1}{4}$ of £20

(b) $\dfrac{1}{5}$ of 30 kg

(c) $\dfrac{3}{4}$ of £32

(d) $\dfrac{4}{5}$ of 90 kg

(e) $\dfrac{5}{7}$ of 49 kg

(f) $\dfrac{3}{8}$ of 20 m

(g) $\dfrac{3}{5}$ of £36

(h) $\dfrac{7}{10}$ of 42 m

6. Calculate:

(a) $\dfrac{1}{2} \times \dfrac{1}{4}$

(b) $\dfrac{3}{8} \times \dfrac{1}{5}$

(c) $\dfrac{2}{3} \times \dfrac{3}{5}$

(d) $\dfrac{6}{7} \times \dfrac{2}{3}$

(e) $\dfrac{4}{5} \times \dfrac{3}{4}$

(f) $\dfrac{4}{7} \times \dfrac{3}{5}$

(g) $\dfrac{1}{2} \times \dfrac{3}{4}$

(h) $\dfrac{4}{9} \times \dfrac{3}{7}$

(i) $\dfrac{1}{8} \times \dfrac{4}{5}$

7. Calculate:

(a) $\dfrac{1}{2} \div \dfrac{1}{3}$

(b) $\dfrac{3}{4} \div \dfrac{8}{9}$

(c) $\dfrac{3}{5} \div \dfrac{4}{5}$

(d) $\dfrac{7}{10} \div \dfrac{1}{2}$

(e) $\dfrac{3}{4} \div \dfrac{3}{5}$

(f) $\dfrac{5}{9} \div \dfrac{7}{8}$

(g) $\dfrac{6}{7} \div \dfrac{2}{3}$

(h) $\dfrac{4}{7} \div \dfrac{3}{4}$

(i) $\dfrac{5}{6} \div \dfrac{2}{3}$

8. Calculate:

    (a) $1\dfrac{1}{2} \times \dfrac{3}{4}$ (b) $3\dfrac{1}{2} \times \dfrac{2}{7}$ (c) $1\dfrac{1}{4} \times \dfrac{2}{3}$

    (d) $1\dfrac{1}{2} \times \dfrac{1}{4}$ (e) $2\dfrac{1}{2} \times \dfrac{3}{4}$ (f) $1\dfrac{2}{3} \times \dfrac{4}{5}$

9. Calculate:

    (a) $1\dfrac{1}{2} \div \dfrac{3}{4}$ (b) $3\dfrac{1}{2} \div \dfrac{1}{2}$ (c) $2\dfrac{1}{4} \div \dfrac{2}{3}$

    (d) $3\dfrac{1}{2} \div \dfrac{1}{4}$ (e) $4\dfrac{1}{2} \div \dfrac{4}{5}$ (f) $3\dfrac{1}{4} \div \dfrac{2}{3}$

10. Calculate:

    (a) $1\dfrac{1}{2} \times \dfrac{3}{4}$ (b) $3\dfrac{1}{2} \times 1\dfrac{4}{7}$ (c) $\left(1\dfrac{1}{3}\right)^2$

11. Calculate:

    (a) $3\dfrac{3}{4} \div 1\dfrac{1}{2}$ (b) $3\dfrac{1}{2} \div 1\dfrac{1}{4}$ (c) $3\dfrac{1}{3} \div 1\dfrac{3}{7}$

12. Calculate:

    (a) $\dfrac{4}{7} + 1\dfrac{3}{4}$ (b) $2\dfrac{1}{2} \times \dfrac{3}{7}$ (c) $5\dfrac{1}{4} - 3\dfrac{1}{6}$

    (d) $6\dfrac{1}{2} \div 1\dfrac{6}{7}$ (e) $1\dfrac{1}{2} \times 2\dfrac{2}{3}$ (f) $2\dfrac{2}{3} - 1\dfrac{5}{8}$

9.2 Fractions in Context

In this section we consider the use of fractions in various contexts, and how to use the fraction key on a calculator.

Example 1

There are 600 pupils in a school. How many school lunches must be prepared if:

(a) $\dfrac{3}{4}$ of the pupils have school lunches,

(b) $\dfrac{2}{3}$ of the pupils have school lunches?

Solution

(a) $\dfrac{3}{4}$ of $600 = \dfrac{3}{4} \times 600$ or $\dfrac{3}{4}$ of $600 = \dfrac{3}{\underset{1}{\cancel{4}}} \times \overset{150}{\cancel{600}}$

$$= \dfrac{1800}{4}$$

$$= 450 \text{ lunches}$$ $= 450 \text{ lunches}$

(b) $\dfrac{2}{3}$ of $600 = \dfrac{2}{3} \times 600$ or $\dfrac{2}{3}$ of $600 = \dfrac{2}{\underset{1}{\cancel{3}}} \times \overset{200}{\cancel{600}}$

$$= \dfrac{1200}{3}$$

$$= 400 \text{ lunches}$$ $= 400 \text{ lunches}$

Example 2

The diagram opposite shows a rectangle.

(a) Calculate its *perimeter*.

(b) Calculate its *area*.

$2\dfrac{1}{4}$ m

$1\dfrac{1}{3}$ m
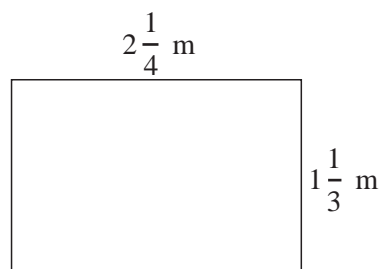
Solution

$$\text{Perimeter} = 2\dfrac{1}{4} + 1\dfrac{1}{3} + 2\dfrac{1}{4} + 1\dfrac{1}{3}$$

$$= 2\dfrac{3}{12} + 1\dfrac{4}{12} + 2\dfrac{3}{12} + 1\dfrac{4}{12}$$

$$= 6\dfrac{14}{12}$$

$$= 7\dfrac{1}{6} \text{ m}$$

$\text{Area} = 2\dfrac{1}{4} \times 1\dfrac{1}{3}$ or $\text{Area} = 2\dfrac{1}{4} \times 1\dfrac{1}{3}$

$$= \dfrac{9}{4} \times \dfrac{4}{3} \qquad\qquad\qquad = \dfrac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \dfrac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}$$

$$= \dfrac{36}{12} \qquad\qquad\qquad\qquad = 3 \text{ m}^2$$

$$= 3 \text{ m}^2$$

Example 3

A loaf of bread requires $\dfrac{3}{4}$ kg of flour. How many loaves can be made from

$6\dfrac{1}{2}$ kg of flour?

Solution

$$
\begin{aligned}
6\frac{1}{2} \div \frac{3}{4} &= \frac{13}{2} \div \frac{3}{4} \\[2mm]
&= \frac{13}{2} \times \frac{4}{3} \\[2mm]
&= \frac{52}{6} \\[2mm]
&= 8\frac{4}{6} \\[2mm]
&= 8\frac{2}{3}
\end{aligned}
$$

8 loaves can be made.

Many calculators have a key marked $\boxed{a\,^b\!/_c}$, which can be used to enter fractions.

Pressing 2 $\boxed{a\,^b\!/_c}$ 3 produces the display $\boxed{\quad 2 \; \lrcorner \; 3 \quad}$ which represents the

fraction $\dfrac{2}{3}$.

Pressing 4 $\boxed{a\,^b\!/_c}$ 7 $\boxed{a\,^b\!/_c}$ 9 produces the display $\boxed{\; 4 \; \lrcorner \; 7 \; \lrcorner \; 9 \;}$, which

represents $4\dfrac{7}{9}$.

Note that you must write the fractions in their correct form, and not just copy the display.

(Some calculator displays may be different from this example – check the instruction booklet for *your* calculator.)

Exercises

1. Use your calculator to find answers for the following, making sure that they are written in the correct way:

(a) $\dfrac{1}{4} + \dfrac{3}{7}$

(b) $\dfrac{5}{7} - \dfrac{1}{3}$

(c) $\dfrac{3}{4} \div \dfrac{1}{9}$

(d) $\dfrac{1}{2} \div \dfrac{1}{6}$

(e) $\dfrac{3}{4} \times \dfrac{7}{8}$

(f) $\dfrac{4}{5} \times \dfrac{3}{8}$

(g) $1\dfrac{1}{2} \times 7$ (h) $2\dfrac{1}{2} \times \dfrac{3}{4}$ (i) $1\dfrac{5}{7} + 4\dfrac{2}{3}$

(j) $1\dfrac{1}{2} \div 1\dfrac{2}{3}$ (k) $6\dfrac{1}{4} \div \dfrac{3}{4}$ (l) $5\dfrac{1}{2} - 3\dfrac{2}{5}$

2. (a) Enter the fraction $\dfrac{6}{8}$ and then press the $\boxed{=}$ key on your calculator. Describe what happens.

   (b) Enter the fraction $\dfrac{8}{6}$ and then press the $\boxed{=}$ key on your calculator. Describe what happens.

   (c) What happens to each of the fractions listed below if you enter it into your calculator and then press the $\boxed{=}$ key:

$$\dfrac{3}{7}, \quad \dfrac{9}{2}, \quad \dfrac{4}{6}, \quad \dfrac{6}{4}, \quad \dfrac{10}{3}, \quad \dfrac{3}{10}$$

3. Calculate the area and perimeter for each of the rectangles below:

   (a) $\dfrac{2}{3}$ m   (b) $2\dfrac{3}{5}$ m

   $\dfrac{1}{2}$ m $1\dfrac{1}{4}$ m
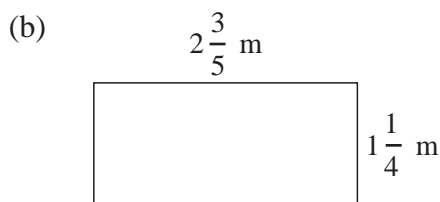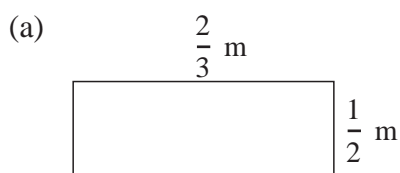
4. A school has 800 pupils. The Headteacher decides to send a questionnaire to $\dfrac{2}{5}$ of the pupils. How many pupils receive a questionnaire?

5. A firm that makes floppy discs knows that $\dfrac{1}{20}$ of the discs they produce have faults. How many faulty floppy discs would you have if you bought:

   (a) 100 discs,   (b) 80 discs,   (c) 2000 discs ?

6. A cake recipe requires $\dfrac{3}{8}$ kg of flour. How many cakes could be made with:

   (a) 3 kg flour,   (b) 6 kg flour,   (c) $\dfrac{2}{3}$ kg flour,

   (d) 1 kg flour,   (e) $1\dfrac{1}{2}$ kg flour,   (f) $1\dfrac{1}{3}$ kg flour.

7. The rectangle opposite has an area of $2\dfrac{3}{5}$ cm^2.

   What is the length, x, of the rectangle?

$\dfrac{3}{4}$ cm

x

8. Sheets of paper are $\dfrac{1}{80}$ cm thick. Calculate the height of a pile of paper that contains:

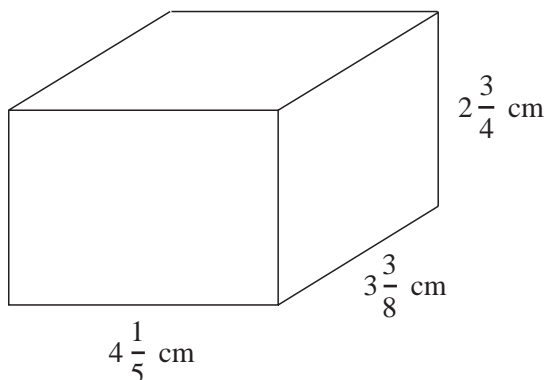   (a)   40 sheets,   (b)   120 sheets,

    (c)   70 sheets,   (d)   140 sheets.

    How many sheets would there be in a pile of paper $4\dfrac{1}{2}$ cm high?

9. A bottle contains $1\dfrac{2}{5}$ litres of orange squash. To make one drink, $\dfrac{1}{200}$ of a litre of squash is needed.

    How many drinks can be made from the bottle of squash?

10. Calculate the volume of the following cuboid:

$2\dfrac{3}{4}$ cm

$3\dfrac{3}{8}$ cm

$4\dfrac{1}{5}$ cm

9.3 Conversion of Fractions and Percentages

To convert a *fraction* to a *percentage, multiply* by 100.

To convert a *percentage* to a *fraction, divide* by 100 or multiply by $\dfrac{1}{100}$.

Example 1

Convert the following fractions to percentages:

(a) $\dfrac{17}{100}$ (b) $\dfrac{9}{10}$ (c) $\dfrac{3}{5}$

(d) $\dfrac{3}{4}$ (e) $\dfrac{1}{3}$ (f) $\dfrac{1}{8}$

Solution

(a) $\dfrac{17}{100} \times 100 = \dfrac{1700}{100}$ or $\dfrac{17}{\underset{1}{\cancel{100}}} \times \overset{1}{\cancel{100}} = 17\%$

             $= 17\%$

(b) $\dfrac{9}{10} \times 100 = \dfrac{900}{10}$ or $\dfrac{9}{\underset{1}{\cancel{10}}} \times \overset{10}{\cancel{100}} = 90\%$

             $= 90\%$

(c) $\dfrac{3}{5} \times 100 = \dfrac{300}{5}$ or $\dfrac{3}{\underset{1}{\cancel{5}}} \times \overset{20}{\cancel{100}} = 60\%$

             $= 60\%$

(d) $\dfrac{3}{4} \times 100 = \dfrac{300}{4}$ or $\dfrac{3}{\underset{1}{\cancel{4}}} \times \overset{25}{\cancel{100}} = 75\%$

             $= 75\%$

(e) $\dfrac{1}{3} \times 100 = \dfrac{100}{3}$

             $= 33\dfrac{1}{3}\%$

(f) $\dfrac{1}{8} \times 100 = \dfrac{100}{8}$

             $= 12\dfrac{4}{8}$

             $= 12\dfrac{1}{2}\%$

Example 2

Convert these percentages to fractions:

(a)   30%   (b)   80%   (c)   45%

(d)   6%   (e) $16\dfrac{1}{2}\%$ (f) $62\dfrac{1}{2}\%$

Solution

(a) $30\% = \dfrac{30}{100}$

             $= \dfrac{3}{10}$

(b) $80\% = \dfrac{80}{100}$

$= \dfrac{8}{10}$

(c) $45\% = \dfrac{45}{100}$

$= \dfrac{9}{20}$

(d) $6\% = \dfrac{6}{100}$

$= \dfrac{3}{50}$

(e) $16\dfrac{1}{2}\% = 16\dfrac{1}{2} \times \dfrac{1}{100}$

$= \dfrac{33}{2} \times \dfrac{1}{100}$

$= \dfrac{33}{200}$

(f) $62\dfrac{1}{2}\% = 62\dfrac{1}{2} \times \dfrac{1}{100}$

$= \dfrac{125}{2} \times \dfrac{1}{100}$

$= \dfrac{125}{200}$

$= \dfrac{5}{8}$

Example 3

A football team is based on a squad of 20 players. In one season 8 players are shown a red or yellow card.

(a)   What percentage of the squad is shown a red or yellow card?

(b)   What percentage of the squad is *not* shown a red or yellow card?

Solution

(a) $\dfrac{8}{20} \times 100 = \dfrac{800}{20}$ or $\dfrac{8}{1\!-\!20} \times \overset{5}{\cancel{100}} = 40\%$

$= 40\%$

(b) $100 - 40 = 60\%$

Exercises

1. Convert the following percentages to fractions:

   (a) 50%   (b) 75%   (c) 40%

   (d) 25%   (e) 20%   (f) 10%

   (g) 8%   (h) 58%   (i) 36%

   (j) 64%   (k) 76%   (l) 12%

2. Convert the following fractions to percentages:

   (a) $\dfrac{7}{10}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$

   (d) $\dfrac{3}{4}$ (e) $\dfrac{7}{20}$ (f) $\dfrac{6}{25}$

   (g) $\dfrac{19}{20}$ (h) $\dfrac{17}{25}$ (i) $\dfrac{3}{5}$

   (j) $\dfrac{1}{5}$ (k) $\dfrac{11}{20}$ (l) $\dfrac{7}{50}$

3. Convert the following percentages to fractions:

   (a) $12\dfrac{1}{2}\%$ (b) $66\dfrac{2}{3}\%$ (c) $33\dfrac{1}{3}\%$

   (d) $14\dfrac{1}{2}\%$ (e) $18\dfrac{1}{2}\%$ (f) $4\dfrac{1}{4}\%$

4. Convert these fractions to percentages:

   (a) $\dfrac{1}{8}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{3}{8}$

   (d) $\dfrac{47}{200}$ (e) $\dfrac{61}{200}$ (f) $\dfrac{2}{3}$

5. In a class of 25 pupils there are 8 individuals who play in the school hockey team. What percentage of the class play in the hockey team?

6. Halim scores 32 out of 80 in a test. Express his score as a percentage.

7.   An athlete has completed 250 m of a 400 m race. What percentage of the distance has the athlete run?

8.   A double decker bus has 72 seats; there are 18 empty seats on the bus.

    (a)   What percentage of the seats are empty?

    (b)   What percentage of the seats are being used?

9.   Andy buys a bag of 12 apples at a supermarket; there are 4 bruised apples in the bag.

    (a)   What percentage of the apples are bruised?

    (b)   What percentage of the apples are *not* bruised?

10. Jason took 4 tests at school and his results are given below:

Science	60	out of	80
Maths	75	out of	100
English	38	out of	50
French	28	out of	40

    (a)   Express his score for each test as a percentage.

    (b)   Write down his average percentage score for the 4 tests.

9.4 Finding Percentages

In this section we revise finding percentages of quantities.

Example 1

Calculate 20% of £120.

Solution

$$20\% \text{ of } £120 = \frac{20}{100} \times 120$$

$$= \frac{2}{10} \times 120$$

$$= £24$$

Example 2

Calculate 75% of 48 kg.

Solution

$$75\% \text{ of } 48 \text{ kg} = \frac{75}{100} \times 48$$

$$= \frac{3}{4} \times 48$$

$$= 36 \text{ kg}$$

Value Added Tax (VAT) is added to the price of many products; in the UK it is currently $17\frac{1}{2}\%$. An interesting way to calculate $17\frac{1}{2}\%$ is to use the fact that $17\frac{1}{2} = 10 + 5 + 2\frac{1}{2}$; this is illustrated in the next example.

Example 3

A bike costs £180 before VAT is added. How much VAT must be added to the cost of the bike, if VAT is charged at $17\frac{1}{2}\%$?

Solution

$$10\% \quad \text{of } £180 = £18$$

$$5\% \quad \text{of } £180 = £9$$

$$2\frac{1}{2}\% \quad \text{of } £180 = £4.50$$

$$17\frac{1}{2}\% \quad \text{of } £180 = £18 + £9 + £4.50$$

$$= £31.50$$

Exercises

1. Calculate:

   (a) 50% of £22   (b) 10% of 70 m   (c) 25% of £60

   (d) 30% of 80 m   (e) 60% of £40   (f) 90% of 50 kg

   (g) 75% of £30   (h) 25% of 6 kg   (i) 30% of 32 kg

   (j) 16% of £40   (k) 70% of 8 m   (l) 35% of £20

2. Use the method of Example 3 to calculate the VAT that must be added to the
   following prices at a rate of $17\frac{1}{2}$%:

   (a) £200   (b) £300   (c) £40

   (d) £30   (e) £28   (f) £38

3. (a) Calculate $17\frac{1}{2}$% of £25

   (b) Describe the most sensible way to give your answer.

4. Calculate $17\frac{1}{2}$% of the following amounts, giving your answers to a sensible
   degree of accuracy:

   (a) £15   (b) £75   (c) £7

5. Use a method similar to Example 3 to calculate 15% of £120.

6. A computer costs £900, but $17\frac{1}{2}$% VAT must be added to this price.

   (a) Calculate $17\frac{1}{2}$% of £900.

   (b) Calculate the total cost of the computer.

7. A company employs 240 staff. The number of staff is to be increased by 20%.
   How many *new* staff will the company employ?

8. A bike costs £186. The price is to be reduced by $33\frac{1}{3}$% in a sale.

   (a) Calculate how much you would save by buying the bike in the sale.

   (b) How much would you pay for the bike in the sale?

9. In a school there are 280 pupils in Year 7. 85% of these pupils go on a trip to
   Alton Towers. How many pupils go on the trip?

10. Alec scores 75% on a test with a maximum of 56 marks. How many marks
    does Alec score in the test?

9.5 Increasing and Decreasing Quantities by a Percentage

When increasing or decreasing by a percentage there are two possible approaches: one is to find the actual increase or decrease and to add it to, or subtract it from, the original amount. The second approach is to use a simple multiplication. For example, to increase by 20%, multiply by 1.2. We can illustrate this by considering a price, say £p , that increases by 20%.

The increase is $£p \times \dfrac{20}{100}$ $= £0.2\,p$

so the new price is

$$£p + £0.2p = £(1 + 0.2)p$$

$$= £1.2p$$

and we can see that a 20% increase is equivalent to multiplying by 1.2 to get the new price.

Note that

$$100\% + 20\% = 120\% \Rightarrow \frac{120}{100} = 1.2$$

Similarly, a decrease of 20% is equivalent to

$$100\% - 20\% = 80\% \Rightarrow \frac{80}{100} = 0.8$$

i.e. a multiplication by 0.8.

Example 1

The price of a jar of coffee is £2.00. Calculate the new price after an increase of 10%.

Solution

10% of £2.00 $= \dfrac{10}{100} \times 2$ or $100\% + 10\% = 110\%,$

$= £0.2$ so multiply by 1.1

New price $= 2 + 0.2$ New price $= 1.1 \times £2$

$= £2.20$ $= £2.20$

Example 2

In a sale, the price of a TV is reduced by 40%. What is the sale price if the original price was £170.

Solution

$$40\% \text{ of } £170 = \frac{40}{100} \times 170 \qquad \text{or} \qquad 100\% - 40\% = 60\%,$$

$$= £68 \qquad\qquad\qquad\qquad \text{so multiply by } 0.6$$

$$\text{Sale price} = 170 - 68 \qquad\qquad\qquad \text{Sale price} = 0.6 \times 170$$

$$= £102 \qquad\qquad\qquad\qquad\qquad = £102$$

Example 3

Jared earns £24 each week by working in a shop. His wages are to be increased by 5%. How much will he then earn each week?

Solution

$$5\% \text{ of } £24 = \frac{5}{100} \times 24 \qquad \text{or} \qquad 100\% + 5\% = 105\%,$$

$$= £1.20 \qquad\qquad\qquad\qquad \text{so multiply by } 1.05$$

$$\text{New wages} = 24 + 1.20 \qquad\qquad\qquad \text{New wages} = 1.05 \times 24$$

$$= £25.20 \qquad\qquad\qquad\qquad\qquad = £25.20$$

Exercises

1. Add 10% to:
   (a) £40   (b) £136   (c) £262

2. Reduce the following prices by 20%:
   (a) £50   (b) £92   (c) £340

3. (a) Increase 40 m by 30%   (b) Increase £60 by 5%
   (c) Increase £66 by 20%   (d) Increase 80 kg by 40%
   (e) Increase £1000 by 30%   (f) Decrease £60 by 25%
   (g) Reduce 70 kg by 5%   (h) Reduce £90 by 15%
   (i) Increase 40 m by 7%   (j) Increase £18 by 4%

4. A computer costs £600. In a sale there is a 20% discount on the price of the item. Calculate the sale price of the computer.

5. A shopkeeper increases all the prices in his shop by 4%. What is the new price of each of the items below? Give your answers to the nearest penny.

Box of chocolates	£3
Bag of flour	75p
Packet of sweets	50p
Tin of beans	20p
Can of drink	45p

6. A CD player costs £90. In a sale the price is reduced by 25%. Calculate the sale price.

7. A certain type of calculator costs £8. A teacher buys 30 of these calculators for her school and is given a 20% discount. How much does she pay in total?

8. Add $17\frac{1}{2}$% VAT to the following prices, giving your answers to the nearest pence:

   (a) £400   (b) £22   (c) £65

9. The population of a town is 120 000. What is the total population of the town after a 5% increase?

10. Hannah invests £800 in a building society. Every year 5% interest is added to her money.

    (a) Explain why, after 2 years she has £882 in her account.

    (b) How much money does she have after 5 years? (Give your answer to the nearest pence.)

11. Andrew has £100 to invest in a building society. At the end of each year, 10% interest is added to his investment.

    (a) What is the multiplier that can be used each year to calculate the new amount in the account?

    (b) Show that the multiplier for 2 years is 1.21.

    (c) What is the multiplier for *n* years?

    (d) How many years does it take to *double* the £100 investment?

9.6 Finding the Percentage Increase and Decrease

When a quantity increases, we can find the percentage increase using this formula:

$$\text{Percentage } \textit{increase} \ = \ \frac{\text{increase}}{\text{original amount}} \times 100$$

Similarly,

$$\text{Percentage } \textit{decrease} \ = \ \frac{\text{decrease}}{\text{original amount}} \times 100$$

Example 1

The price of a drink increases from 40p to 45p. What is the percentage increase?

Solution

$$
\begin{aligned}
\text{Increase} \ &= \ 45\text{p} - 40\text{p} \\
&= \ 5\text{p}
\end{aligned}
$$

$$
\begin{aligned}
\text{Percentage increase} \ &= \ \frac{5}{40} \times 100 \\
&= \ \frac{25}{2} \\
&= \ 12.5\%
\end{aligned}
$$

Example 2

The number of pupils in a school increases from 820 to 861. Calculate the percentage increase.

Solution

$$
\begin{aligned}
\text{Increase} \ &= \ 861 - 820 \\
&= \ 41 \text{ pupils}
\end{aligned}
$$

$$
\begin{aligned}
\text{Percentage increase} \ &= \ \frac{41}{820} \times 100 \\
&= \ 5\%
\end{aligned}
$$

Example 3

Although the lion is thought of as an African animal, there is a small population in India and elsewhere in Asia. The number of lions in India decreased from 6000 to 3900 over a 10-year period. Calculate the percentage decrease in this period.

Solution

Decrease = 6000 − 3900

= 2100 lions

Percentage decrease = $\dfrac{2100}{6000} \times 100$

= 35%

Example 4

The price of cheese, per kg, is increased from £3.26 to £3.84. What is the percentage increase?

Solution

Increase = £3.84 − £3.26

= £0.58

Percentage increase = $\dfrac{0.58}{3.26} \times 100$

= 17.8% to 1 decimal place

Note: You might find it easier to work through the example in pence, but note that *all* quantities must be expressed in pence.

Increase = $(384 - 326)$p

= 58p

Percentage increase = $\dfrac{58}{326} \times 100$

= 17.8% to 1 decimal place

Example 5

In a sale, the price of a bike is reduced from £180 to £138. Calculate the percentage reduction in price, correct to 1 decimal place.

Solution

Reduction = 180 − 138

= £42

Percentage reduction = $\dfrac{42}{180} \times 100$

= 23.3% to 1 decimal place.

Exercises

1. The price of a school lunch increases from £1.40 to £1.54. Calculate the percentage increase in the price.

2. A television priced at £500 is reduced in price to £400 in a sale. Calculate the percentage reduction in the price of the television.

3. The price of a car increases from £8000 to £8240. What is the percentage increase in the price of the car?

4. A shopkeeper buys notepads for 60p each and sells them for 80p each. What percentage of the selling price is profit?

5. The value of an antique clock increases from £300 to £345. Calculate the percentage increase in the value of the clock.

6. The number of books in a school library is increased from 2220 to 2354. What is the percentage increase in the number of books?

7. The height of a tomato plant increases from 80 cm to 95 cm. Calculate the percentage increase in the height, correct to 1 decimal place.

8. The price of a bus fare is reduced from 55p to 40p. Calculate the percentage reduction in the price of the bus fare, correct to 1 decimal place.

9. The mass of a person on a diet decreases from 75 kg to 74 kg. Calculate the percentage reduction in their mass, correct to 3 significant figures.

10. Jasmine invests £250 in a building society. After the first year her account contains £262.50. After the second year it contains £280.88. Calculate the percentage increase of the amount in her account:

    (a) during the first year,

    (b) during the second year,

    (c) over the two years.

    Give your answers correct to 2 decimal places.

9.7 Reverse Percentage Calculations

The process of adding a percentage to a quantity can be reversed.

For example, if the cost of a portable TV is £141 including $17\frac{1}{2}$% VAT, the cost *before* adding the VAT can be found. The multiplier in this example is 1.175, as the price is made up of $100\% + 17.5\% = 117.5\%$, which is equivalent to multiplying by

$$\frac{117.5}{100} = 1.175$$
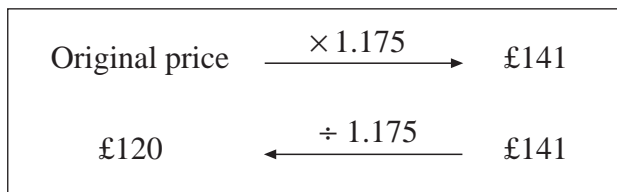
Example 1

Jane's salary was increased by 10% to £9350. What was her original salary?

Solution

$100\% + 10\% = 110\%,$

$$\text{which} = \frac{110}{100} = 1.1$$

Therefore Jane's original salary would have been multiplied by 1.1 to give £9350.

So to calculate her original salary, divide by 1.1.

Example 2

In a sale, the price of a video recorder is reduced by 22% to £218.40. How much money would you save by buying the video recorder in the sale?

Solution

$$100\% - 22\% \quad = \quad 78\%$$
$$= \quad \frac{78}{100}$$
$$= \quad 0.78$$

The original price would have been multiplied by 0.78 to get the sale price. So divide by 0.78 to find the original price.

Original price $\xrightarrow{\quad \times 0.78 \quad}$ £218.40

£280 $\xleftarrow{\quad \div 0.78 \quad}$ £218.40

$$\begin{aligned}
\text{Saving} \quad &= \quad \text{Original price} - \text{Sale price} \\
&= \quad £280 - £218.40 \\
&= \quad £61.60
\end{aligned}$$

Example 3

The cost of an order, including VAT at $17\frac{1}{2}\%$, is £274.95.

Calculate the cost of the order *without* VAT.

Solution

Original cost $\xrightarrow{\quad \times 1.175 \quad}$ £274.95

£234 $\xleftarrow{\quad \div 1.175 \quad}$ £274.95

Cost of the order without VAT is £234.00.

Exercises

1. In a sale the prices of all the clothes in a shop are reduced by 20%. Calculate the original prices of the items below:

Item	Sale Price
Jeans	£36
Coat	£56
Shirt	£14

2. The price of a car is increased by 4% to £12 480. What was the original price?

3. The amount that Jason earns for his paper round is increased by 2% to £21.93 per week. How much *extra* money does Jason now get each week?

4. A special value packet of breakfast cereal contains 25% more than the standard packet. The special value packet contains 562.5 grams of cereal. How much does the *standard* packet contain?

5. The bill for repairing a computer is £29.38 which includes VAT at $17\frac{1}{2}\%$.

   What was the bill before the VAT was added?

6. The height of a plant increases by 18%, to 26 cm. Calculate the original height of the plant, correct to the nearest cm.

7. A 3.5% pay rise increases Mr Smith's annual salary to £21 735. What was his original salary?

8. The price of a bike in a sale is £145. If the original price has been reduced by $12\frac{1}{2}\%$, what was the original price? (Give your answer to the nearest pence.)

9. Alice carries out an experiment to record how quickly plants grow. One plant increases in height from 12.0 cm to 13.8 cm in one week. A second plant increases by the same percentage to 16.1 cm. What was the original height of the second plant?

10. James buys a computer. The seller reduces the price by 30% and adds VAT at 17.5%. If James pays £1551 for the computer, what was its original price? (Give your answer to the nearest pence.)